Designing fractional orderPID for car suspension systems using PSO Algorithm

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Abstract
The suspension system of the car has a crucial effect on the comfort of traveling andcontrolling the vehicle because the body of the car is assembled on it and transfer the forces by the road to the body. In this study the implementation of FOPID controller based on PSO algorithm on the 1/4 active suspension system was investigated through the non-linear hydraulic actuator. The working principle of hydraulic suspension system that sometimes is called hydropenomatic is based on the compressibility principles of gases and non-compressibility of liquids. Investigation of the given acceleration to the pssengers and deviation of suspension shows that the suggested controlling structure has made more ease for the aboard. The results of simulation of the system regarding a non-flat road as the entrance, proves the ideal operation of closed ring system.

Key words: suspension system of the vehicle, PSO algorithm, hydraulic actuator, FOPID controller

1- Introduction

The need to have a comfort ride and the security of cars has made many car industries to use the active suspension system. Such suspension systems that are controlled by electronic tools, improve the movement quality and its safety. The suspension systems of car are classified into three classes based on quality. The inactive suspension systems in which the pneumatic or hydraulic actuators are used for perfect controlling of vibration of the car [1]. In the active suspension systems the pneumatic or hydraulic actuators are used to have perfect controlling of the amount of car vibration that are put in parallel to the springs and low springs and the suitable strategy is applied using the information from vibration. Four important parameters that should be taken into consideration in designing each suspension system are as follows: the ease of travelling, the movement of body of the automobile, stability on the road, the movement of suspension components of the system.

Of course, no suspension system can optimize all the four mentioned parameters simultaneously. But it provides the ease of the passengers while keeping the stability by creating a compromise among them. It is possible to gain an optimized promise among the parameters in the active suspension system. Designing the active suspension systems of the vehicle is the topic of many modern researches in car industry in the world while the non-linear behavior of hydraulic actuator of the system is not considered. On the other hand, the practical experiments have shown the importance of the non-linear behavior of the actuator in identifying the optimal compromise in the suspension system of the car based on the empirical results [2,3,4] in 1/4 model of vehicle that is illustrated in figure 1. In this model, only the vertical movement of the body on one wheel is considered and in many cases is used to prove the given controlling strategy [3, 5, 6, and 7].

In [9] a PID controller system is designed for the suspension system of the car. The slide controller mode is designed in [10] for the active suspension. The slide controlling mode is used to weaken the vibration. The active suspension using the information observed previously and controlling the previous model is given in [11]. In this method, the predictor controlling model is expressed by using the previous observed information to control the active suspension systems. Designing MPC considers clearly all the limitations on the modes, control, and output variables.

2- Mathematical premises

In spite of the complexities in fractionalcalculus, in the recent decades by improvements in chaosareas and the close relationship of fractals with fractionalcalculus has caused that interest in applying it to be increased. Fractional calculus has a more domain than the integral derivation. If instead of the order of integral derivation, the fractional rank is used the fractional calculus should be used to solve the fractional derivation and integral.

The derivation or integral taker operator is shown by ${}_aD_t^{\alpha}$. This actuator is a symbol that is used to take the derivation and fractional integral. α is the symbol of derivation for positive numbers and it shows the integral for negative numbers. Definitions that are usually used for fractional derivation are as follows: Granvalld-letinkove, Riemann -Liovil, and Caputo and are expressed as follows:

Granvald- Letinkov:

Definition of Granvald- Letinkov is as follows:

$${}_aD_t^{\alpha} f(t) = \lim_{h\to 0} h^{-\alpha} \sum_{j=0}^{[t-\alpha/h]} (-1)^j \binom{\alpha}{j} f(t-jh), \quad (1)$$

The upper limit sum, in the above equation should go toward the infinity and $\frac{t-a}{h}$ has this quality (a, t are the upper and down limits of the derivation taking). The formula of Granvald- Letinkov can be used to take the fractional integral. The simplest change for using this formula in taking the integral is using it for $\alpha < 0$. In this mode, we should make $\binom{-\alpha}{m}$ definable by gamma (γ) function.

Definition of Riemann–Liovil [8] second definition is definition of RL that is used as the simplest and easiest definition and is as follows:

$$ {}_aD_t^{\alpha} f(t) = \frac{1}{\Gamma(n-\alpha)} \frac{d}{dt} \int_a^t \frac{f(\tau)}{(t-\tau)^{\alpha-n+1}} d\tau, \quad (2) $$

That $n-1 < \alpha < n$, and also $\Gamma(.)$ is the famous γ function. Its lapillus transformation is as follows:

$$ \int_0^{\infty} e^{-st} {}_0D_t^{\alpha} f(t) dt = s^{\alpha} F(s) \qquad \alpha \le 0 $$

$$ \int_0^{\infty} e^{-st} {}_0D_t^{\alpha} f(t) dt = s^{\alpha} F(s) - \sum_{k=0}^{n-1} s^k {}_0D_t^{\alpha-k-1} f(t)\big|_{t=0} \qquad n-1 < \alpha \le n \in N \quad (3) $$

3- Description of the model

The model of suspension system using the method of Newton- Oilier has been defined as follows [3.5].

$$ M_s\ddot{Z}_s = -K_s(Z_s - Z_u) - B_s(\dot{Z}_s - \dot{Z}_u) + F_A - F_f \quad (4) $$
$$ M_s\ddot{Z}_s = -K_s(Z_s - Z_u)_{+F_f} + B_s(\dot{Z}_s - \dot{Z}_u) - K_t(Z_u - Z_r) - F_A \quad (5) $$

In which:

Z_r: $the turbulant input of the road$

$F_{A:}$:the hydraulic actuatorpower

F_f :friction power of hydraulic actuator

U: the input

In this model, M_s is the mass of a quarter of the body; M_u is the mass of a wheel and its suspension tools. The wheel of the automobile is modeled as a spring with the coefficient of $K_t$and it is supposed that its attenuation is little. The element of power generator is considered as a hydraulic actuator in active suspension system. This element causes the compensation of chaos resulted from unevenness of the road by creating a changing power. Movement of needlevalve is controlled by a bladed valve with a direct current. Based on the researches,

The dynamic of servo valve that includes bladed and needle-like valve has in fact three poles resulted from hydraulic, mechanic and electric systems. However the dominant pole of the system is related to its hydraulic system and consequently is modeled after a number one system that is realized by experiments.

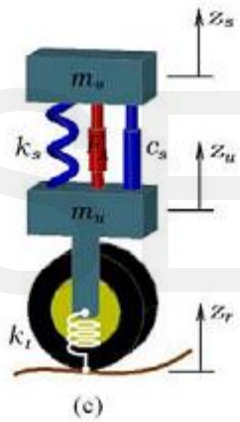

Figure 1. 1/4 suspension system of vehicle

The non-linear relationship between movement of bladed valve and the power of operator is given in equation (6) [7].

$$F = P_I . A \qquad (6)$$

The non-linear relations in hydraulic actuator are given in equations (7) to (9).

$$\frac{V_t}{4\beta_e}\dot{P}_l = Q_l - C_{tm}P_l - A\left(\dot{Z}_s - \dot{Z}_u\right) \qquad (7)$$

$$Q_l - C_{d\omega x_v}\sqrt{\frac{P_s - sgn(x_v)P_l}{\rho}} \qquad (8)$$

$$F_A = Aa\left[C_{d\omega x_v}\sqrt{\frac{P_s - sgn(x_v)P_l}{\rho}} - C_{tm}P_l - A\left(\dot{Z}_s - \dot{Z}_u\right)\right] \qquad (9)$$

The friction power of the actuator is considerable and is modeled empirically with the curve of speed according to a homogenous approximation from Signum function and the relations are expressed in (10) and (11).

$$i_f\left|\dot{Z}_s-\dot{Z}_u\right|>0.01\frac{m}{s}\,then\;F_f=\mu\,sgn\left(\dot{Z}_s-\dot{Z}_u\right) \qquad (10)$$

$$i_f\left|\dot{Z}_s-\dot{Z}_u\right|<0.01\frac{m}{s}\,then\;F_f=\mu\,sgn\left(\frac{\dot{Z}_s-\dot{Z}_u}{0.01}\frac{\pi}{2}\right) \qquad (11)$$

To apply the controlling methods it is necessary to extract the mode space model of the system or the transferring function that is dominant on it. The mode space model of the system is given here. It should be noted that system output here is the acceleration of the body of the vehicle. So the mode variables are selected as equations (12) to (16).

$$\dot{x_1}=Z_s-Z_u \qquad (12)$$

$$x_2=\dot{Z_s}(10)$$

$$x_3=Z_u-Z_r \qquad (13)$$

$$x_4=\dot{Z_u} \qquad (14)$$

$$x_5=P_1-P_2 \qquad (15)$$

$$x_6=x_v \qquad (16)$$

And the equations of system mode are shown as equations (17) to (23).

$$\dot{x_1}=x_2-x_4 \qquad (17)$$

$$\dot{\dot{x_2}}=\frac{1}{M_s}(-K_sx_1-B_s(x_2-x_4)+Ax_5-F_f) \qquad (18)$$

$$\dot{\dot{x_3}}=x_4-\dot{z}_4 \qquad (19)$$

$$\dot{x}_4=\frac{1}{M_u}(K_sx_1-B_s(x_2-x_4)-K_tx_3-+Ax_5-F_f) \qquad (20)$$

$$\dot{x}_5=-Bx_5-\alpha A(x_2-x_4)+\gamma x_6\sqrt{p_s-sgn(x_6)x_5} \qquad (21)$$

$$\dot{x}_6=\frac{1}{\tau}(-x_6+u) \qquad (22)$$

$$\mathrm{y}=Ax_5-F_f \qquad (23)$$

The amounts of system parameters are also shown empirically in table 1.

Parameters	Symbols	Quantities
Borty mass	m_s	290kg
Wheel mass	m_u	59kg
Stiffness of the body	K_s	16kN/m
Stiffness of the wheel	K_t	19kN/m
Stiffness of the damper	B_s	1kN.s/m

Area of piston	A	$3359e^4m^2$
Supply pressure	P_s	10342500 pa
$4\beta/V_1$	A	$4515e^{13}N/m^5$
ac_{tm}	B	1.00
ac_dw	γ	$1549e^9$ NL$(m^{\frac{5}{2}}kg^{\frac{1}{2}})$

Table 1.The numerical amounts of system parameters.

Definitions of parameters used in the system:
Hydraulic fluid density= ρ
The entrance of servo valve = U
Operator volume=V_t
Subsidence coefficient=C_{tm}
Discharge coefficient= C_d
The width of controller valve= W
Pressure of the source=P_s

Figure 2 (a) shows that designer is only devoted to designing 4 kinds of controllers PID, PD, PI, P while regarding figure(b), with changing the amount of integral taking (λ) and the amount of taking derivation (μ) between 0 and 1 it is possible to design fractional controllers $PI^{\lambda}D^{\mu}$ in addition to designing integral controllers.

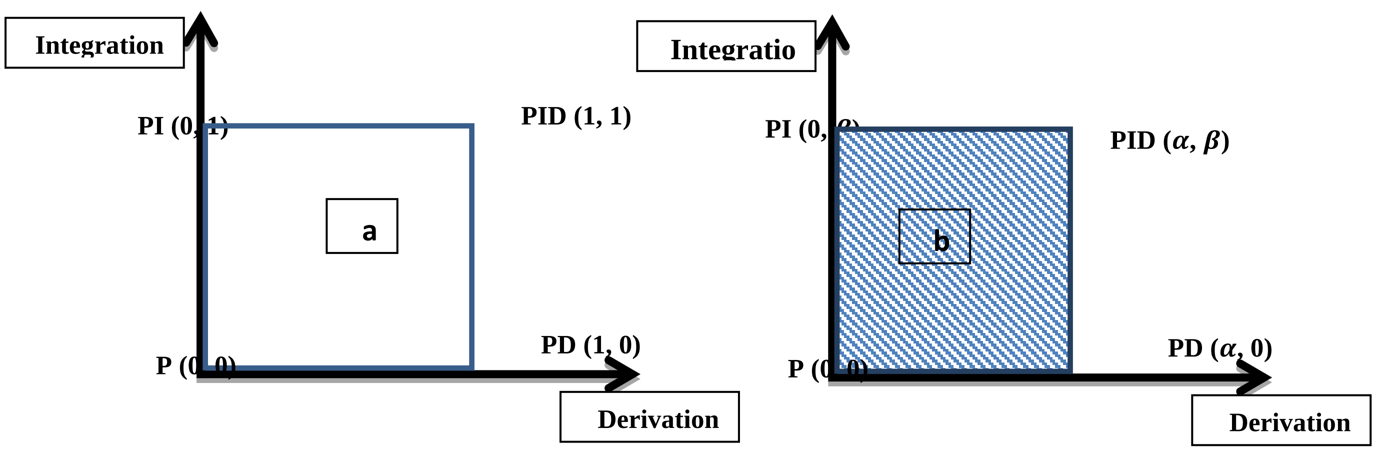

Figure2: Integer and fractional order controllers.

Although PID controllers of integerorders are applied widely in industry, but these controllers don't have a high performance needed for high ranking systems and also fractional systems. So, using fractional controllers in industryis a very new field of research.

Particle Swarm Optimization BASED TUNING OF THE $PI^{-\lambda}$ D $^{\delta}$ CONTROLLER GAINS

Particle swarm optimization, first developed by Kennedy and Eberhart [4], is one of the modern heuristic algorithms. It was inspired by the social behavior of bird and fish schooling, and has been found to be robust in solving continuous nonlinear optimization problems.

This algorithm is based on the following scenario: a group of birds are randomly searching food in an area and there is only one piece of food. All birds are unaware where the food is, but they do know how far the food is at each time instant. The best and most effective strategy to find the food would be to follow the bird which is nearest to the food. Based on such scenario, the PSO algorithm is used to solve the optimization problem.

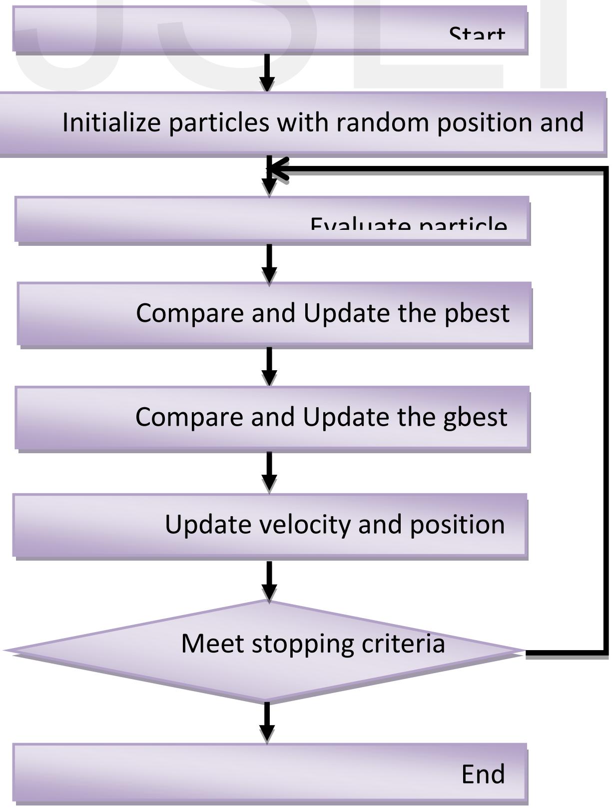

Figure 3 The PSO algorithm procedure

In PSO, each single solution is a "bird" in the search space; this is referred to as a "particle". The swarm is modeled as particles in a multidimensional space, which have positions and velocities. These particles have two essential capabilities: their memory of their own best position and knowledge of the global best. Members of a swarm communicate good positions to each other and adjust their own position and velocity based on good positions according to (24).

$$\begin{aligned} v(k+1)_{i,j} &= w.v(k)_{i,j} + c_1 r_1 \left(gbest - x(k)_{i,j}\right) \\ &\quad + c_2 r_2 \left(pbest_j - x(k)_{i,j}\right) \qquad (24) \\ x(k+1)_{i,j} &= x(k)_{i,j} + v(k)_{i,j} \end{aligned}$$

Where

$V_{i,j}$ Velocity of particle i and dimension j

$X_{i,j}$ Position of particle i and dimension j

c_1, c_2 Acceleration constants

w Inertia weight factor

r_1, r_2 Random numbers between 0 and 1

$pbest$ Best position of a specific particle

$gbest$ Best particle of the group

1. Initialize a group of particles including the random positions, velocities and accelerations of particles.

2. Evaluate the fitness of each particle.

3. Compare the individual fitness of each particle to its previous pbest. If the fitness is better, update the fitness as pbest.

4. Compare the individual fitness of each particle to its previous gbest. If the fitness is better, update the fitness as gbest.

5. Update velocity and position of each particle according to (8).

6. Go back to step 2 of the process and keep repeating until some stopping condition is met.

Although PSO has been relatively recently developed, there already exist several applications based on the PSO algorithm. One application of PSO algorithms is the optimized particle filter. A particle filter aims to estimate a sequence of hidden parameters based only on the observed data. This can help for example in removing noise and by this improving the measured data. The PSO can be merged into the particle filter for optimization.

The basic particle filter is suboptimal in the sampling step, by applying PSO to particle filter, particle impoverishment problem is avoided and estimation accuracy is improved.

The PSO algorithms can also be applied in the job shop scheduling [5]. Job shop scheduling is an optimization problem in which ideal jobs are assigned to resources at particular times. In this problem, several jobs of varying size are given, which need to be scheduled on different identical machines, while trying to minimize the total length of the schedule. By converting the job shop scheduling problem to PSO we can search the optimal solution for job shop scheduling. Through using PSO for the job shop problem we can find a scheduling solution at least as good as the best known solution. Moreover PSO generally results in a better solution than GA while its implementation and manipulation is easier.

Fig 4 illustrates the block structure of the FOPID controller optimizing process with PSO. All parameters of the FOPID controllers are updated at every final time (t f). It should be noted that the values of an element in the particle may exceed its reasonable range. In this case, inspired from practical requirements and from the papers focusing on tuning the parameters of FOPID in application of the different systems, the lower bound of the FOPID parameters are zero and their upper bounds are set.

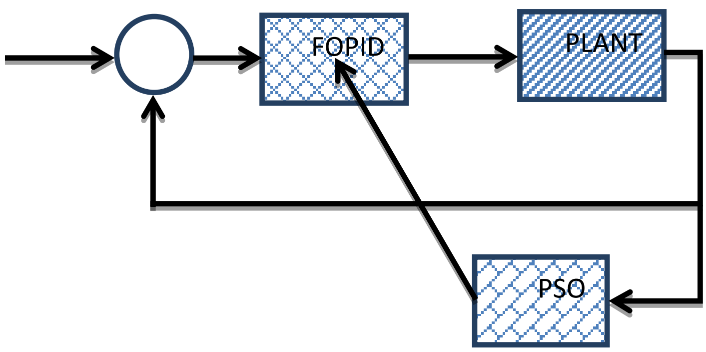

Figure4. Tuning process of the FOPID controller parameters with PSO

Figure4. Controller optimizing of fractional order with genetic algorithm.

4- **Simulation:** in figure 5 the suspension system simu-link is observed. This simulated model is in the issue with controller of fractional order. PID

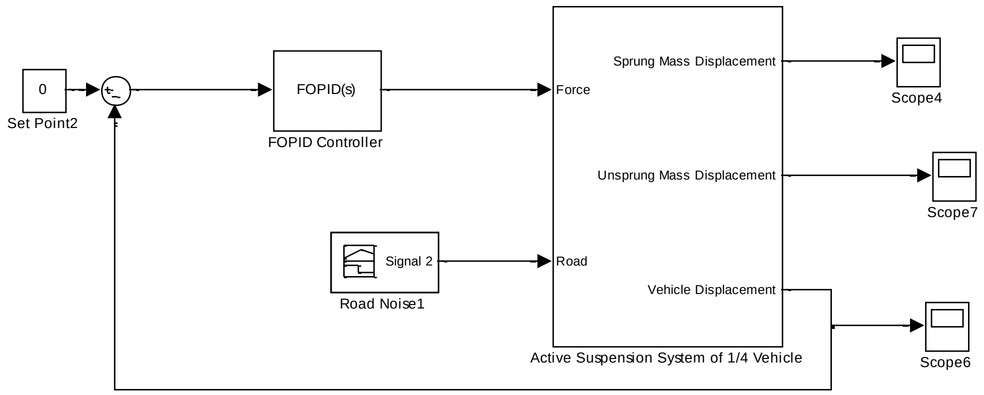

Figure5: simulation of 1/4 suspension system in vehicle

In figure6, the road entrance that is considered an uneven road is seen. The road entrance has both ups and downs. The road has ups t seconds 2 and 6 and it has down at second 4.

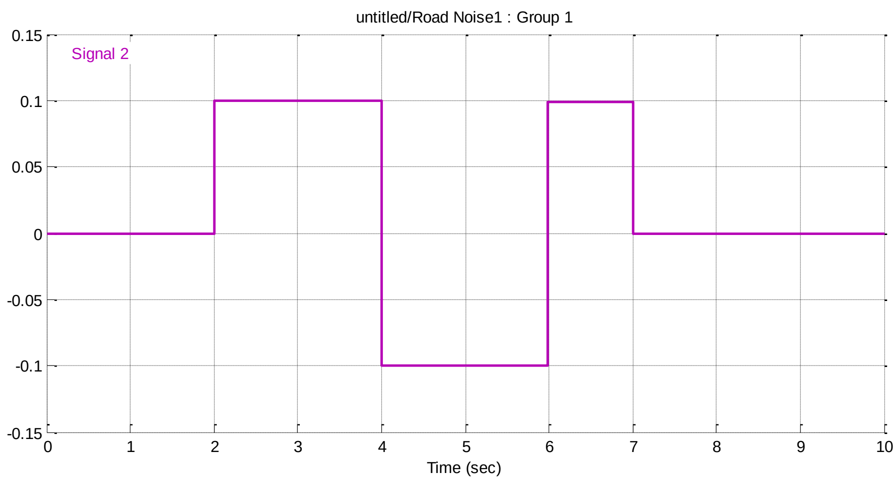

Fig.6. the road noise

It is clear that the gained results that are obtained based on the amounts at table 2 for the controller FOIPD could provide the safety and comfort to a large extent.

	k_p	k_i	k_d	v_i	v_d

FOPID parameter	1.0147	1.7723	9.2305	0.2433	0.9337

As it is seen in figures 7 and 8, the controller could reduce the displacement of suspension system of the body of the car to more than half.

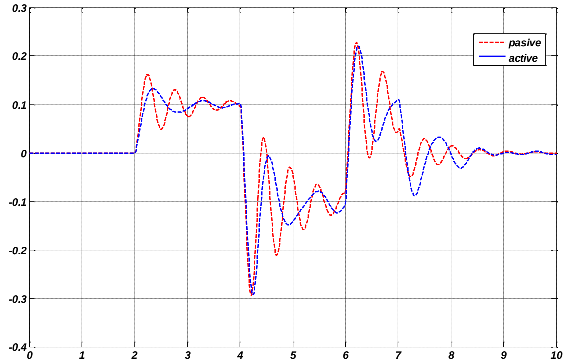

Fig. 7: Displacement of the vehicle

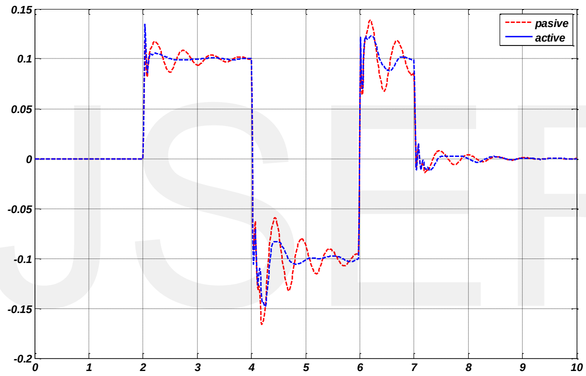

Fig. 8: Displacement of the body

In figure 9, displacement of the wheels of the vehicle is shown in two modes of without control and PID controller. It is seen that this controller could reduce the fluctuations of the wheel as well.

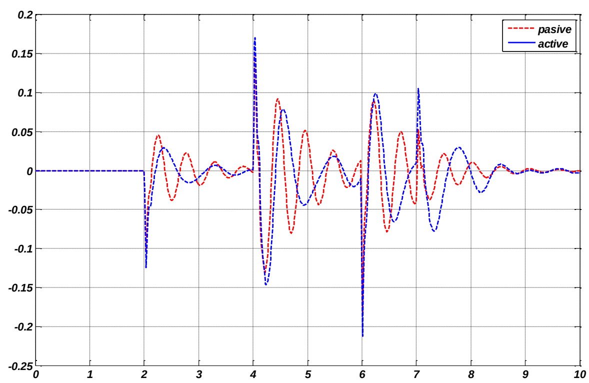

Fig. 9: Displacement of the wheel

The uncertainty is created in this stage.

The uncertainty of the mass is considered as follows:

$$M_{SR} = m_{SR} + \Delta m_{SR}$$

$$M_{wR} = m_{wR} + \Delta m_{wR}$$

The uncertainty is 10% and is considered for $\mathbf{M_{SR}}$ and $\mathbf{M_{wR}}$.

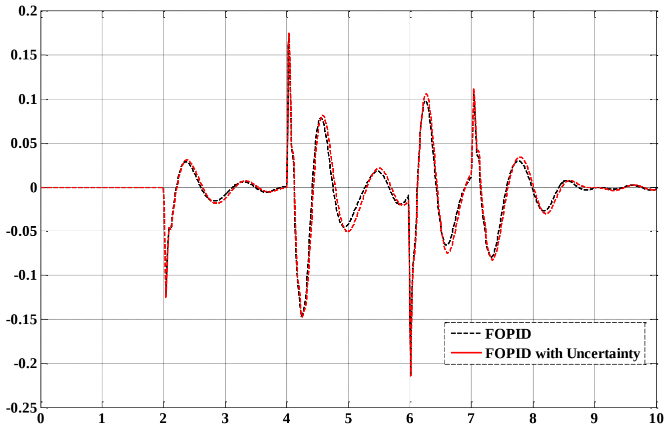

Figure10: reply of the controller of fractional order with uncertainty

In figure 10, the amount of robustness of this controller is shown with uncertainty. Uncertainty of 10% is applied on M_{wR} و M_{SR}. The system reply shows that this designed controller has a perfect performance against uncertainty. This feature is one of the most important characteristics of controllers of fractional order.

5- Conclusion

In this article the genetic algorithm is applied to design the optimized controller FOPID to control active suspension system of the vehicle. The performance of active system comparing to inactive system shows the superiority of this system that can attract engineer' attention. On the other hand, the optimized controller FOPID designed for this system has a suitable response with non-liner hydraulic actuator that could provide the comfort for the passengers to a high extent and reduce the acceleration on the body of the vehicle. It is hoped that these systems can be used in the car manufacturing industries in future.

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